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RADIATION-CONDUCTION INTERACTION OF STEADY STREAMWISE SURFACE TEMPERATURE VARIATIONS ON VERTICAL FREE CONVECTION

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Abstract: We examine how the steady free convective boundary-layer flow induced by a vertical heated surface is affected by the presence of sinusoidal surface temperature variations about a constant mean value with the effect of radiation. The problem is studied using fully numerical techniques. The surface rate of heat transfer eventually alternates in sign with distance from the leading edge, but no separation occurs unless the amplitude of the thermal modulation is sufficiently high. Numerical results are obtained for different values of the physical parameters, the Planck number R_d, and the surface temperature wave amplitude a. It is observed that both the local shear stress and the Nusselt number decrease when values of R_d increase.

INTRODUCTION

In this paper we describe an investigation of the combined effects of surface temperature variations and radiation on the steady boundary-layer flow of a Newtonian fluid from a heated vertical surface. It is well-known that power-law surface temperature distributions (and also power-law surface heat fluxes) give rise to self-similar boundary layer flows [26, 16]. But here we are interested in another form of surface variation, namely, sinusoidal variations about a mean temperature which is held above the ambient temperature of the fluid. As in [14] this type of surface distribution may be taken as a simplified model of the effects of a periodical array of heaters behind or within the heated surface. An accurate analysis of such a configuration requires a detailed examination of the effects of solid conduction within the heated surface, but the aim of the present work is to simplify the problem by imposing a surface temperature distribution. In this way we can determine a large amount of information about the resulting flow using both numerical methods. Various papers have been published which deal with the effects of surface variations. For example, Chiu and Chou [3], Hossain et. al. [22] and Kim [15] have extended these analyses to micropolar fluids, magnetohydrodynamic convection and non-Newtonian convection, respectively. In a series of papers Rees and Pop [8-12] and Rees [13] have also considered a large variety of analogous flows in porous media. Of these, only [13] has been concerned with the effect of sinusoidal surface temperature variations, although in that case the surface variations were spanwise, thereby giving rise to a three-dimensional flow-field.

Radiative convective flows are encountered in many industrial and environmental processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Mathematically the equations for radiative heat transfer with absorption, scattering and emission can be generated by one of two approaches, namely the continuum model or the spectral radiative treatment of a single particle. Details of the derivation of the general equation of radiative heat transfer are provided in the classic monograph by Chandrasekhar [7]. Little is currently known about the boundary layer flows of radiating fluids. The inclusion of conduction-radiation effects in the energy equation, however, leads to a more highly nonlinear partial differential equation. The majority of studies concerned with the interaction of thermal radiation and natural convection were made by Sparrow and Cess [29], Cess [4], Arpaci [1], Cheng and Ozisik [5], Hasegawa et al. [18], and Bankston et al. [2] for the case of a vertical semi-infinite plate. In recent years, Soundalgekar and Takhar [28] have studied radiation effects on free convection flow of a gas past a semi-infinite flat plate using Cogley-Vincentine-Giles equilibrium model (Cogley et al., [6]) and Hossain and Takhar [19] have analyzed the effect of radiation using the Rosseland diffusion approximation which leads to a nonsimilar mixed convective boundary-layer flow of an optically dense viscous incompressible fluid past a heated vertical plate with a uniform free stream velocity and surface temperature. The boundary layer equations were obtained using a group of transformations and they are valid in both the forced convective and free convective limits. The resulting equations were solved using an implicit finite difference method. Recently the problem of natural convection-radiation interaction on boundary layer flows with the Rosseland diffusion approximation been studied by Hossain and Alim [20] and Hossain et al. [21]; and very recently, Hossain and Rees [22] have investigated the effect of radiation-conduction interaction in the mixed convective flow along a slender impermeable vertical cylinder.

The inclusion of radiation terms is complicated and the resulting equations are very difficult to solve. Grief and Habib [17] have shown that, in the optically thin limit, the physical situation can be simplified and they derived an exact solution of the problem of fully-developed radiating laminar convection flow in an infinite vertical heated channel. Their analysis was based on the work by Cogley et al. [6]. In the optically thin limit the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. It was shown by Cogley *et al.* [6] that in optically thin limit for a gray-gas near equilibrium, the following relation holds:

$$\frac{\partial q_r}{\partial y} = 4(T - T_w)\int_0^\infty \kappa_{\lambda w}\left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w d\lambda = 4(T - T_w)I\ ,$$

$$\text{where} \quad I = \int_0^\infty \kappa_{\lambda w}\left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w d\lambda\ .$$

Here q_r is the radiative flux, κ_λ is the absorption coefficient, $e_{b\lambda}$ is the Planck function and the subscript *w*, represents the value of a quantity at the wall. Further simplification may be made concerning the spectral properties of radiating gases, but this is not essential for the present analysis. It should be mentioned that Soundalgekar and Takhar [28] have considered the radiative free convective flow of a optically thin grey-gas past a semi-infinite vertical plate.

But, the Rosseland model is valid for isotropic local intensity and high optical density of the medium and the radiative heat flux is given by

$$q_r = -\frac{16\sigma\nabla T^3}{3(a_R + \sigma_s)}\nabla T$$

where T denotes the temperature, a_R is the Rosseland absorption coefficient, σ_s is the scattering coefficient and σ is the Stefan-Boltzmann constant. The thermal boundary-layer equation can be written as

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left[\left(\frac{16\sigma T^3}{3a_R} + \kappa\right)\frac{\partial T}{\partial y}\right] \quad .$$

The present paper considers in detail how the combined effects of surface radiation and sinusoidal surface temperature profiles in the streamwise direction modify the otherwise self-similar boundary-layer flow. Solutions are presented in terms of the surface rate of heat transfer and shear stress and detailed isotherms are also given. An important feature of the flow is that a near-wall layer develops at large distances down-stream of the leading edge. The numerical evidence suggests that this inner layer decreases in thickness with distance down-steam. A finite difference method was employed in obtaining the numerical solutions. The effect of varying different physical parameters on the local skin-friction and local rate of heat transfer are presented.

Governing equations and boundary-layer analysis

We consider the boundary layer induced by a heated semi-infinite surface immersed in an incompressible Newtonian fluid. In particular, the heated surface is maintained at the steady temperature.

$$T = T_\infty + (T_w - T_\infty)(1 - a\sin(\pi\hat{x}d)) \qquad (1)$$

where T_∞ is the ambient fluid temperature, T_w is the mean-surface temperature which is such that $T_w > T_\infty$, a is the relative amplitude of the surface temperature variations and 2d is the wavelength of the variations. After a suitable non-dimensionalisation the steady two-dimensional equations of motion are given by

$$u_x + v_y = 0 \qquad (2)$$

$$uu_x + vu_y = -p_x + Gr^{-1/2}(u_{xx} + u_{yy}) + \theta \qquad (3)$$

$$uv_x + vv_y = -p_y + Gr^{-1/2}(u_{xx} + v_{yy}) + \theta \qquad (4)$$

$$u\theta_x + v\theta_y = \alpha Gr^{-1/2}\left[\theta_{yy} + \frac{16\sigma}{3\kappa(a + \sigma_s)}\left\{\theta^3\theta_y\right\}_y\right] \qquad (5)$$

where *Gr* is the Grashof number and σ is the Prandtl number. In the derivation of equations (2) the Boussinesq approximation has been assumed. We note that the Grashof number has been based on *d*, half the dimensional wavelength of the thermal waves.

In the equations, *u* and *v* are, respectively, the velocity components in the *x* and *y* directions, *T* is the fluid temperature, ν is the kinematic viscosity, β is the thermal expansion coefficient, α is the thermal diffusivity, κ is the thermal conductivity , *a* is the Rosseland mean absorption coefficient, σ is the Stephan-Boltzman constant, σ_s is the scattering coefficient When the surface temperature is uniform and the Grashof number is very large, the resulting boundary-layer flow is self-similar. But the presence of sinusoidal surface temperature distributions, such as that given be (1), renders the boundary-layer flow non-similar. The boundary-layer equations are obtained by introducing the scaling

$$u = u^*, \quad v = Gr^{-1/4}v^*, \quad x = x^*, \qquad (6)$$

$$y = Gr^{-1/4}y^*, \quad p = Gr^{-1/2}p^*, \quad \theta = \theta^*$$

into equations (2), formally letting *Gr* become asymptotically large and retaining only the leading order terms. Thus we obtain

$$u_x + v_y = 0 \qquad (7)$$

$$uu_x + vu_y = u_{yy} + \theta \tag{8}$$

$$uv_x + vv_y = -p_y + v_{yy} \tag{9}$$

$$u\theta_x + v\theta_y = \alpha Gr^{-1/2}\left[\theta_{yy} + \frac{16\sigma}{3\kappa(a+\sigma_s)}\left\{\theta^3\theta_y\right\}_y\right] \tag{10}$$

where the asterisk superscripts have been omitted for clarity of presentation. Equation (9) serves to define the pressure field in terms of the two velocity components and is decoupled from the other three equations. Therefore, we shall not consider it further. As the equations are two-dimensional we define a stream function ψ, in the usual way.

$$u = \psi_y, \quad v = -\psi_x \tag{11}$$

and therefore, (7) is satisfied automatically. Guided by the familiar self-similar form corresponding to a uniform surface temperature, we use the substitution

$$\psi = x^{3/4} f(\eta, x), \quad \theta = g(\eta, x) \tag{12}$$

where

$$\eta = y / x^{1/4} \tag{13}$$

is the pseudo-similarity variable. Equation (8) and (9) reduce to

$$f''' + g + \tfrac{3}{4} ff'' - \tfrac{1}{2} f'f' + x(f_x f'' - f_x f') \tag{14}$$

$$\frac{1}{\Pr}\left[\left\{1 + \frac{4}{3}R_d\left(1 + (\theta_w - 1)g\right)^3\right\}g'\right]' + \tfrac{3}{4}\, f \tag{15}$$

and the boundary conditions are

$$f = 0, \quad f' = 0, \quad g = 1 + a\sin\pi x \tag{16}$$

at $\eta = 0$ and $f'g \to 0$ as $\eta \to \infty$.

In equations (14)-(16), primes denote derivatives with respect to η.

NUMERICAL SOLUTIONS

The parabolic system of equations (14)-(16), is non-similar and its numerical solution must be obtained using a marching method. The results presented here were obtained using the Keller-box method, introduced by Keller and Cebeci [17] and described in more detail in Cebeci and Bradshaw [18]. After reduction of equations (14)-(15) to first-order form in η, the subsequent second-order accurate discretisation based halfway between the grid points in both the η- and x-directions yields a set of nonlinear difference equations which are solved using a multi-dimensional Newton-Raphson iteration scheme. The results presented in Fig.1 to Fig. 4 are based on uniform grids in both coordinate directions. There were 201 gridpoints lying between η = 0 and η = 20 and 401 between x = 0 and x = 20. We restrict the presentation of our results to the two values of the Prandtl number: Pr. = 0.7 (air) and Pr. = 7 (water).

Figure 1(a) shows the evolution with x of $f''(\eta = 0)$, a scaled surface shear stress, for various values of the temperature wave amplitude, a, and the constant radiation parameter R_d for Pr=0.7. The corresponding rates of surface heat transfer are shown in Figs 1(b). Some aspects of the overall behavior of these curves may be explained by observing that the boundary layer is thinner when the surface temperature is relatively high and thicker when it is low. This arises because relatively high surface temperatures induce relatively large upward fluid velocities with the consequent increase in the rate of entertainment into the boundary layer. This causes, in turn, a thinning of the boundary layer. Thus, we should expect high shear stresses and rates of heat transfer at, or perhaps just beyond, where the surface temperature attains its maximum values. There is an obvious qualitative difference between the curves shown in Fig. 1(a) and those in Fig. 1(b). As x increases, the amplitude of oscillation of the shear stress curves decays slowly, whereas the amplitude of heat transfer curves increases with x. Indeed, the curves in Fig. 1(b) suggest that, whatever the value of a, there will always be a value of x beyond which some part of the rate of the heat transfer curve between successive surface temperature maxima will be positive. This somewhat unusual phenomenon for boundary layer flows may be explained by noting that when relatively hot fluid encounters a relatively cold part of the heated surface the overall heat transfer will be from the fluid into the surface, rather than the other way around.

Figure 2(a) shows the surface shear stress, for various values of the temperature wave amplitude a and constant radiation parameter R_d for Pr=7.0. The corresponding rates of surface heat transfer are shown in Figs 2(b). Same thing is happening over here as above. Besides this shear stress and rates of surface heat transfer are decreasing than for Pr=0.7.
Figure 3(a) presents surface shear stress, for various values of radiation parameter R_d and constant temperature wave amplitude a for Pr=0.7. The corresponding rates of surface heat transfer are shown in Figs 3(b). These figure describes that when the radiation parameter is increasing both the shear stress and rates of surface heat transfer is decreasing. It is mentionable that whenever the value of radiation parameter R_d=0.0 i.e. there is no effect of radiation then these results have an excellent agreement with the result of Rees[14].
Figure 4(a) presents surface shear stress, for various values of radiation parameter R_d and constant temperature wave amplitude a for Pr=7.0. The corresponding rates of surface heat transfer are shown in Figs 3(b). Here we are observing that both the shear stress and rates of surface heat transfer is decreasing more for Pr=7.0 than Pr=0.7.

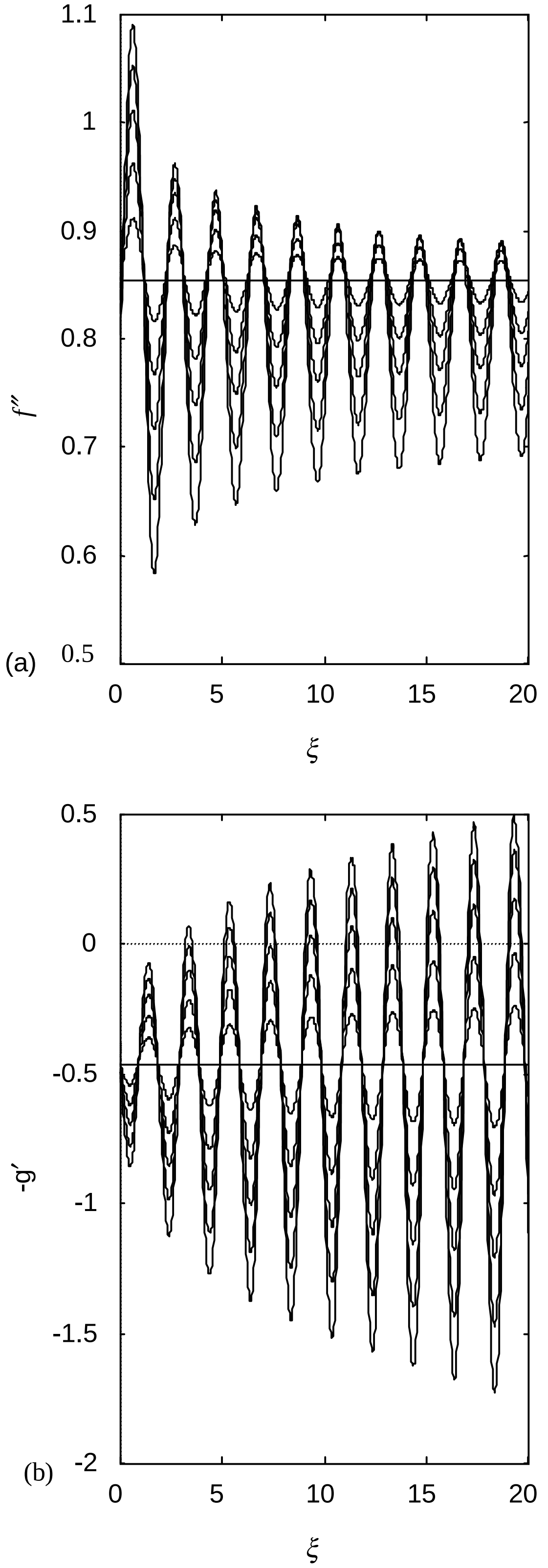

Fig 1: (a) Skin friction and (b) Rate of heat transfer against ξ for a=0.0, 0.2, 0.4, 0.6, 0.8, 1.0 for Pr.=0.7

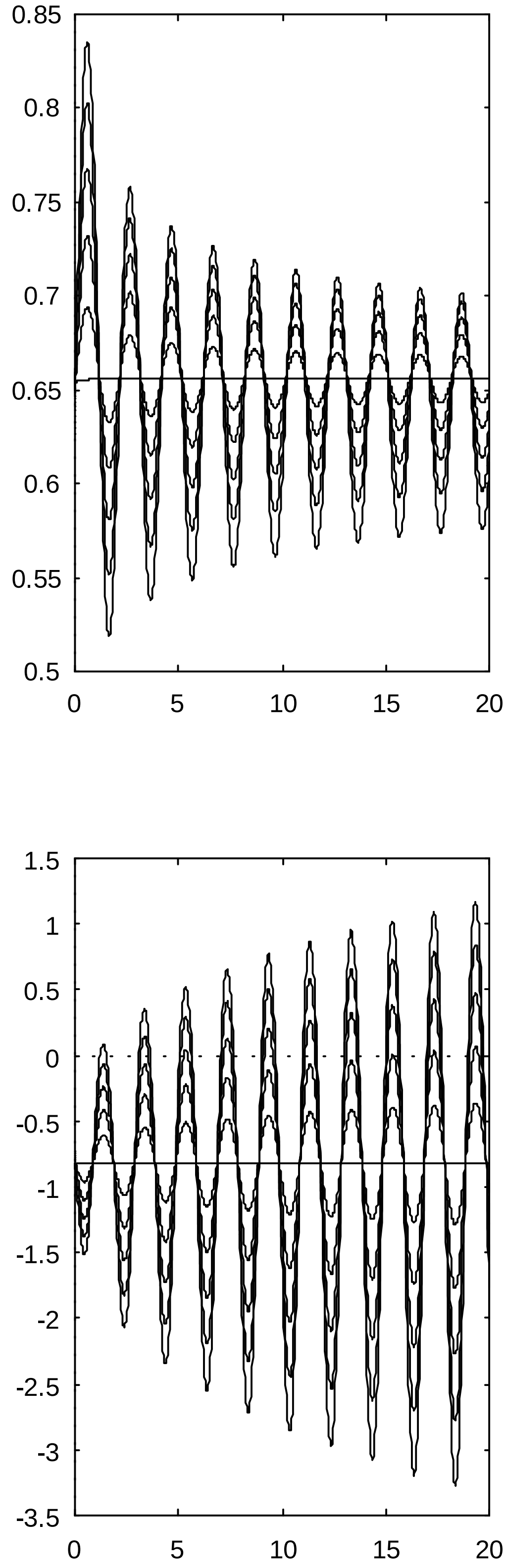

Fig 2: (a) Skin friction and (b) Rate of heat transfer against ξ for a=0.0, 0.2, 0.4, 0.6, 0.8, 1.0 for Pr.=7.0

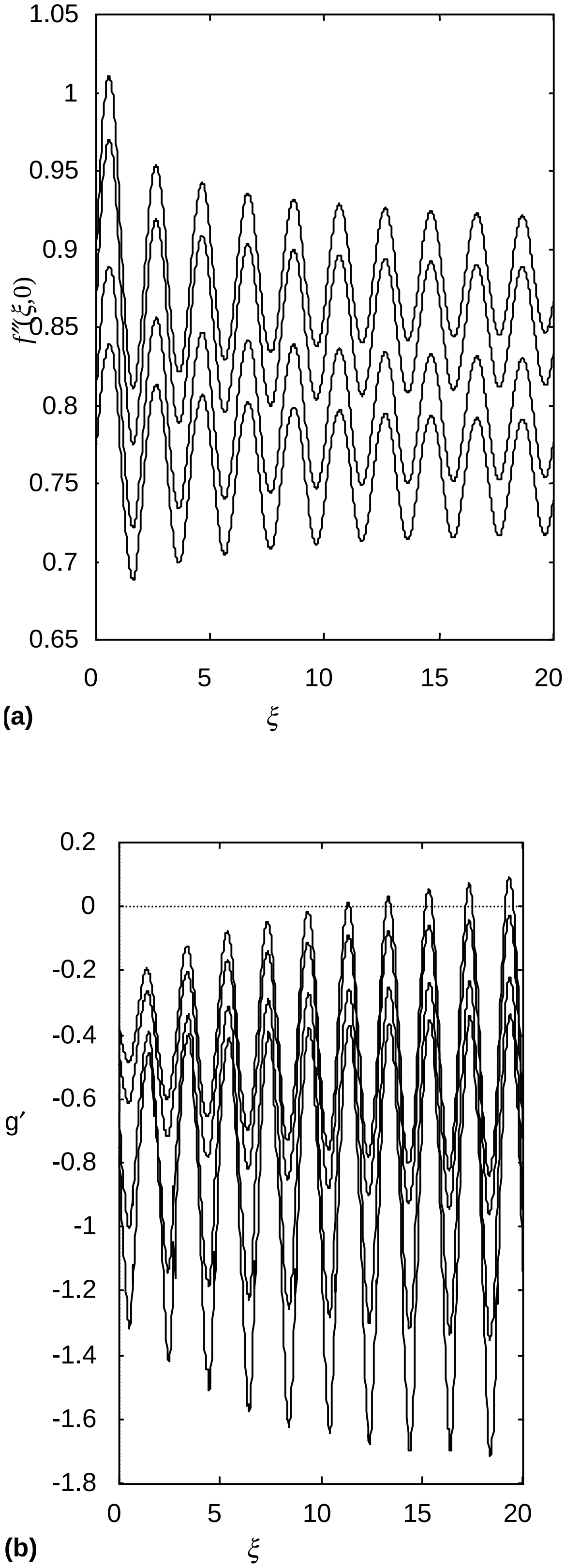

Fig 3: (a) Skin friction and (b) Rate of heat transfer against ξ for R_d=0.0, 1.0, 5.0, 10, for Pr.=0.7

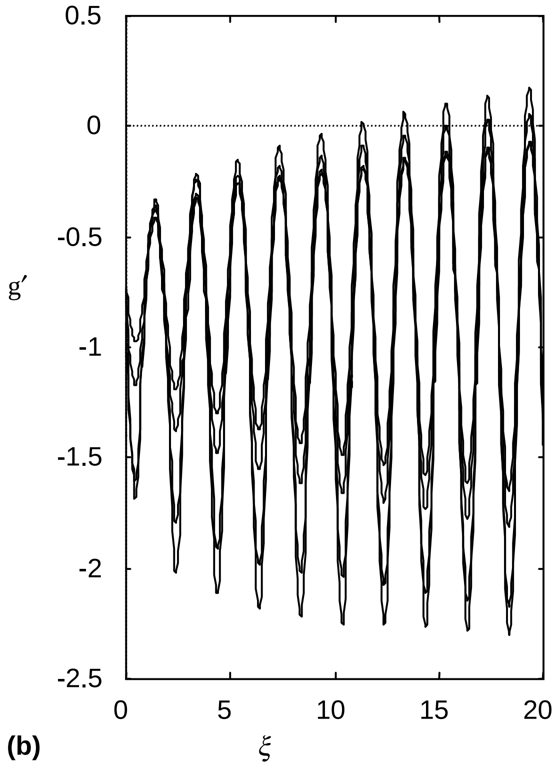

Fig 4: (a) Skin friction and (b) Rate of heat transfer against ξ for R_d=0.0, 1.0, 5.0, 10, for Pr.=7.0

CONCLUSIONS

In this paper the combined effect of radiaton-conduction interaction with steady streamwise surface temperature variations on vertical free convection has been investigated numerically using a finite difference method. The effect of variations in the Plank number, the surface temperature wave amplitude, and the Prandtl number on the shear stress and rate of surface heat transfer have been given. We observed that an increase in the radiation parameter and in Pr leads to increasing values in the skin friction and the rate of heat transfer.

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NOMENCLATURE

a	Surface temperature wave amplitude
a_R	Rosseland mean absorption coefficient
f	Dimensionless velocity function
g	Dimensionless temperature acceleration due to gravity
Gr	Grashof number
Pr	Prandtl number
d	Half the dimensional thermal wave length
R_d	Radiation Parameter
p	Pressure
u,v	Velocity components
T	Temperature of the fluid
T_∞	Temperature of the ambient fluid
x, y	Streamwise and cross stream Cartesian coordinate
ψ	Stream function
σ	Stefan-Boltzman constant
σ_s	Scattering coefficient
β	Coefficient expansion
ν	Kinematic coefficient of viscosity
η	Psuedo-similarity variable
θ	Temperature
θ_w	Surface Temperature parameter
ξ,	Dimensionless x coordinates
ρ	Density of the fluid
T_w	Mean-surface temperature to the wall

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